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Performance evaluation of crossflow compact heat exchangers using finite elements

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Abstract—An analysis of a crossflow compact heat exchanger is carried out using a finite element model. The predictions are in good agreement with analytical solutions available for cases with constant heat transfer coefficients. The analysis is extended to variable heat transfer coefficient cases, and the results compared with constant heat transfer coefficient cases. In addition to thermal analysis, the pressure drops are calculated for all cases.

INTRODUCTION

IN THE last 50 years, a number of reports have been published by various authors about the analysis of single-pass crossflow exchangers. Recently, Baclic and Heggs [1] have argued that all these reports are simply reproductions of Nusselt's results in different forms. Incidentally, in all these derivations, constant heat transfer coefficients (hereafter called CHTC) were assumed throughout the exchanger. Analytical expressions for ε and other performance parameters were evaluated on the basis of Nusselt's model, and they are presented in standard design textbooks [2, 3]. While sizing or rating an exchanger, heat transfer coefficients are evaluated at a bulk mean temperature (either known *a priori* or assumed). In the case of large variations in temperatures from inlet to outlet of a fluid, Kays and London [2] suggested considering the exchanger in parts, in each of which the temperature changes of the fluids are small. Alternatively, they suggested ways to calculate the effective bulk mean temperatures of the fluids, but concluded by saying that they can only be taken as satisfactory guides, not the exact values. Yamashita *et al.* [4] derived the expressions for the performance parameters, with presumed variations in heat transfer coefficients such as linear, exponential, etc., in simple flow arrangements where both the fluids are mixed. A numerical procedure based on the finite element method is developed to analyse crossflow compact heat exchangers. The linear elements which are used in the present paper are equivalent to the finite difference scheme with central differences. The boundary conditions are easily incorporated into the finite element method. If better accuracy is required, the finite element method can be extended to include higher order elements (like quadratic variation of the temperatures in both the fluids). Since the solution with linear elements is closer to analytical solutions, higher order elements are not used. The exchanger where both the fluids are unmixed is analysed. The analysis is used to study the effects of variable heat transfer coefficients (VHTC) on the thermal performance and pressure drop.

FINITE ELEMENT METHOD

The exchanger to be analysed is discretized into a number of smaller exchangers (called strips) as shown in Fig. 1. Every strip consists of a number of pairs of stacks which carry hot and cold fluids as shown in Fig. 2. A pair of stacks marked (A) is separated; this is shown in Fig. 3. This is the basic elemental exchanger for which the finite element equations are derived. The heat transfer in the elemental exchanger is schematically shown in Fig. 4. In the element, the temperatures of the fluids are assumed to vary only along their flow lengths.

The system of equations which govern the heat transfer in the exchanger is

$$q_1 = -\alpha_h(T_h - T_{m,1}) \quad (1)$$

$$C_h \frac{dT_h}{dx} = -\alpha_h(T_h - T_{m,1}) - \alpha_h(T_h - T_{m,4}) \quad (2)$$

NOMENCLATURE

C	heat capacity [kJ kg^{-1} °C^{-1}]
f	fanning friction factor
G'	mass flux [kg s^{-1} m^{-2}]
K_c	sudden contraction coefficient, see ref. [2]
K_e	sudden expansion coefficient, see ref. [2]
N_1, N_2	linear shape functions, defined after equation (7)
NTU	number of transfer units
P	pressure [Pa]
q	enthalpy entering or leaving node [W]
R	ratio of heat capacity rates, C_{min}/C_{max}
S	total heat transfer area [m^2]
S_f	free flow area on one side [m^2]
T	temperature of hot/cold fluid and metal wall [°C]
ΔT_m	true mean temperature difference [°C]
x	flow length along hot fluid [m]
y	flow length along cold fluid [m]
Z	specific volume [m^3 kg^{-1}].

Greek symbols

α	heat transfer coefficient [W m^{-2} K^{-1}]
ε	effectiveness of heat exchanger
σ	ratio of free flow area to frontal area of one side of exchanger
θ	ratio of true mean temperature difference to the inlet temperatures difference, $\Delta T_m/(T_{h,in}-T_{c,in})$.

Subscripts

c	cold fluid
h	hot fluid
i	inlet
1–7	node numbers when used with T or q.

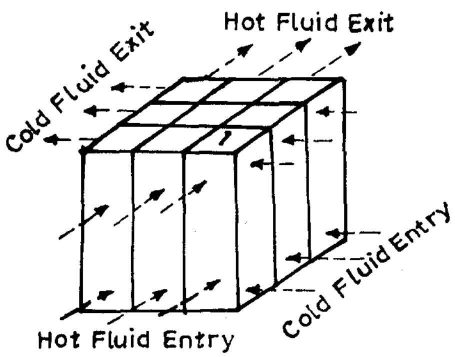

FIG. 1. Discretized exchanger.

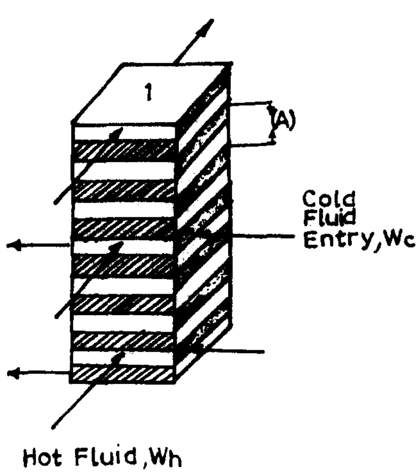

FIG. 2. Flow arrangement in strip 1.

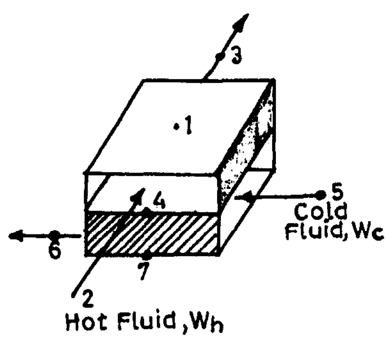

FIG. 3. Flow arrangement in an element with node numbers.

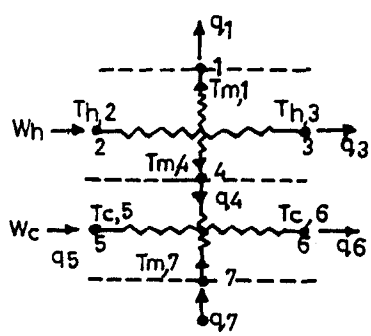

FIG. 4. Schematic representation of heat transfer in the element.

$$\alpha_h(T_h - T_{m,4}) = \alpha_c(T_{m,4} - T_c) \tag{3}$$

$$C_c\frac{dT_c}{dy} = \alpha_c(T_{m,4} - T_c) + \alpha_c(T_{m,7} - T_c) \tag{4}$$

$$q_7 = \alpha_c(T_{m,7} - T_c). \tag{5}$$

The temperatures of the cold (T_c) and hot (T_h) fluids in the element are approximated by a linear variation

$$T_h = N_1 T_{h,2} + N_2 T_{h,3} \quad (6)$$

$$T_c = N_1 T_{c,5} + N_2 T_{c,6} \quad (7)$$

where N_1 and N_2 are shape functions, $N_1 = 1 - x/L$, $N_2 = x/L$ and L is the element length. The boundary conditions to be satisfied are

$$q_2 = C_h T_{h,2} \quad (8)$$

$$q_5 = C_c T_{c,5} \quad (9)$$

where $T_{h,2}$ and $T_{c,5}$ are known boundary conditions for the element. Substituting approximations (6) and (7) into equations (1)–(5) and applying the subdomain collocation procedure to minimize the error over the entire domain, the final set of element matrices can be obtained as

$$\begin{bmatrix} D_h & \frac{-D_h}{2} & \frac{-D_h}{2} & 0 & 0 & 0 & 0 \\ 0 & C_h & 0 & 0 & 0 & 0 & 0 \\ D_h & C_h - D_h & -D_h + C_h & D_h & 0 & 0 & 0 \\ 0 & -D_h & -D_h & 2(D_h + D_c) & -D_c & -D_c & 0 \\ 0 & 0 & 0 & 0 & C_c & 0 & 0 \\ 0 & 0 & 0 & D_c & (C_c - D_c) & -(C_c + D_c) & D_c \\ 0 & 0 & 0 & 0 & \frac{-D_c}{2} & \frac{-D_c}{2} & D_c \end{bmatrix} \begin{Bmatrix} T_{m,1} \\ T_{m,2} \\ T_{m,3} \\ T_{m,4} \\ T_{c,5} \\ T_{c,6} \\ T_{m,7} \end{Bmatrix} = \begin{Bmatrix} -q_1 \\ q_2 \\ 0 \\ 0 \\ q_5 \\ 0 \\ q_7 \end{Bmatrix} \quad (10)$$

where

$$D_h = \alpha_h A_h, \quad D_c = \alpha_c A_c.$$

The element matrices for other pairs of stacks in the strip (Fig. 2) are evaluated and assembled into a global matrix. The term q_7 on the right-hand side of equation (10) gets cancelled when the adjacent element is assembled. It remains on the right-hand side of the global matrix only for the bottom pair of stacks. Similarly, q_1 remains only for the top pair of stacks. If the top and bottom surfaces are insulated, then q_1 for the top pair of stacks and q_7 for the bottom pair become zero. The final sets of simultaneous equations are solved after incorporating the known boundary conditions (inlet temperatures or heat quantities). The outlet temperatures of strip 1 (Fig. 1) will be the inlet temperatures for adjacent strips 2 and 4. Thus by marching in a proper sequence from 1 to 9, the temperature distribution in the exchanger is obtained.

ILLUSTRATION

Application of the method to constant heat transfer coefficient (CHTC) case [5]

If the fluid properties and heat transfer coefficients are assumed to be constant throughout the exchanger, then the heat capacity rates C_h, C_c and the heat transfer coefficients α_h, α_c are evaluated at the bulk mean temperatures of the fluids. They are used in the calculation of element matrices. If the bulk mean temperatures are not known *a priori*, the iteration is started with assumed outlet temperatures. The new outlet temperatures obtained from the calculation are compared with previous values. The iterations are continued until the two consecutive outlet temperatures agree. To establish the prediction capability of the method for CHTC cases, a few sample processes are considered. The characteristics of the surfaces used are given in Table 1. The flow quantities, the inlet conditions of the fluids and dimensions of exchangers used for the different cases are listed in Table 2. The temperature distribution is obtained as explained in previous sections. The temperature distributions on the outlet surfaces for the heat exchanger are averaged to get representative outlet temperatures for hot and cold fluids. The total heat (Q) transferred is evaluated by summing the heat transferred from every element. The representative true mean temperature difference ΔT_m is calculated by averaging true, mean temperatures existing in all the elemental exchangers.

Table 1. Characteristics of the surfaces and other constant quantities

Quantity	Fluid 1 (hot)	Fluid 2 (cold)
Surface index	1/8–16.00(D)	1/8–19.82(D)
Plate spacing (mm)	3.17	2.54
Plate thickness (mm)	0.152	0.152
Conductivity of plates ($\mathrm{W\,m^{-1}\,K^{-1}}$)	202.5	202.5
Fluids	Air	Air
Number of stacks	43	43

Table 2. The flow quantities, inlet conditions† and exchanger dimensions for the different cases

No.	Flow quantity ($kg\ s^{-1}$) Hot	Cold	Inlet temperature (°C) Hot	Cold	Dimensions (flow length) (m) Hot	Cold
1	0.575	0.544	355.5	56.1	0.216	0.139
2	0.756	0.756	648.8	65.5	0.244	0.183
3	0.756	0.504	648.8	65.5	0.244	0.183
4	1.008	0.756	648.8	65.5	0.244	0.183
5	0.756	0.378	648.8	65.5	0.244	0.183
6	1.008	0.504	648.8	65.5	0.305	0.244
7	1.008	0.756	648.8	65.5	0.305	0.244
8	1.008	1.008	648.8	65.5	0.305	0.244
9	1.26	1.008	648.8	65.5	0.305	0.244

† Refer to Table 4 for inlet pressure conditions.

The performance parameters are calculated from

$$\varepsilon = Q/C_{\min}(T_{\mathrm{h,in}} - T_{\mathrm{c,in}}) \tag{11}$$

$$NTU = Q/\Delta T_{\mathrm{m}} C_{\min} \tag{12}$$

$$\theta = \Delta T_{\mathrm{m}}/(T_{\mathrm{h,in}} - T_{\mathrm{c,in}}) \tag{13}$$

$$R = C_{\min}/C_{\max}. \tag{14}$$

The performance factors numerically evaluated for the sample problems are compared in Table 3 with ε and NTU determined from the published analytical formula [2, 3].

The ratio of heat transferred to the pumping power required is an important factor in economizing an exchanger. Pumping power depends upon the pressure drop required to pump the fluid at the required velocity. Hence, it is necessary to estimate the pressure drop along with thermal performance. The pressure drops in the fluids are evaluated using [2]

$$\frac{\Delta P}{P_{\mathrm{in}}} = \frac{G^2 Z_1}{2 g_{\mathrm{c}} P_{\mathrm{in}}}\left[\underbrace{(1 + K_{\mathrm{c}} - \sigma^2)}_{\text{entrance}} + \underbrace{2\left(\frac{Z_2}{Z_1} - 1\right)}_{\text{acceleration}} + \underbrace{f\frac{S Z_{\mathrm{m}}}{S_{\mathrm{f}} Z_1}}_{\text{friction}} - \underbrace{(1 - \sigma^2 - K_{\mathrm{e}})\frac{Z_2}{Z_1}}_{\text{exit}}\right] \tag{15}$$

with the *f*s evaluated at the bulk mean temperatures of the fluids obtained from thermal performance analysis. The entrance and exit loss terms in equation (15) exist only for the elements located on the entrance and exit face of the exchanger. The outlet pressures evaluated in sample problems are given in Table 4. Comparison of the results from the present method with the analytical results in Tables 3 and 4 shows that the present method is capable of predicting accurately the temperature distribution and performance parameters of compact crossflow exchangers.

Application of the method to variable heat transfer coefficient (VHTC) case

The heat transfer coefficient will vary throughout the exchanger due to the entry length effect and large changes in fluid temperatures from inlet to outlet. The entry length effects are not considered in the present analysis. The solution procedure to obtain the temperature distribution in the exchanger is similar to that of the CHTC case except that the heat transfer coefficient and other properties vary from element to element, depending on their local bulk mean temperatures. To maintain the continuity of energy between the strips, specific heats C_p of both the fluids (air) are maintained constant throughout the exchanger. The viscosity and thermal conductivity are calculated as functions of the local bulk mean temperatures of the elements. For air, both viscosity and thermal conductivity increase with temperature. The properties are taken from ref. [7]. Since the local bulk

Table 3. Comparison of performance parameters for constant and variable heat transfer coefficient cases

No.	R	NTU_{a}	NTU_{c}	NTU_{v}	ε_{a}	ε_{c}	ε_{v}	θ_{a}	θ_{c}	θ_{v}
1	0.93	2.806	2.806	2.765	0.689	0.687	0.689	0.246	0.245	0.249
2	0.96	3.466	3.466	3.396	0.713	0.712	0.741	0.208	0.205	0.218
3	0.64	4.666	4.666	4.482	0.852	0.852	0.882	0.183	0.183	0.197
4	0.72	3.796	3.796	3.678	0.797	0.796	0.824	0.211	0.209	0.224
5	0.48	5.721	5.721	5.433	0.923	0.925	0.956	0.162	0.162	0.176
6	0.48	7.321	7.321	6.908	0.948	0.950	0.992	0.129	0.129	0.144
7	0.72	5.467	5.467	5.254	0.850	0.849	0.879	0.156	0.155	0.167
8	0.96	4.405	4.405	4.321	0.747	0.744	0.776	0.171	0.169	0.179
9	0.77	4.747	4.747	4.607	0.815	0.815	0.846	0.173	0.172	0.184

a, analytical result; c, numerical results with CHTC; v, numerical results with VHTC.

Table 4. Pressure† drop comparison for case 1, given in Table 2

No.	Fluid 1 Inlet	Fluid 1 Outlet Constant	Fluid 1 Outlet Variable	Fluid 2 Inlet	Fluid 2 Outlet Constant	Fluid 2 Outlet Variable	Fluid 2 Pressure drop error (%)
1	389.556	385.143	385.281	55.158	30.130	33.163	13.7
2	389.556	382.592	382.316	103.422	77.359	77.359	0.0
3	389.556	382.040	381.834	103.422	90.660	89.425	8.8
4	389.556	377.421	377.421	103.422	75.153	75.015	0.0
5	389.556	381.696	381.489	103.422	95.286	93.976	13.8
6	389.556	379.765	379.558	103.422	91.080	89.080	13.9
7	389.556	380.524	380.248	103.422	78.462	77.152	4.9
8	389.556	381.006	380.799	172.370	152.030	151.134	4.2
9	389.556	376.387	376.387	172.370	150.927	149.686	5.4

† Pressure in kPa.

mean temperatures are not known *a priori* for an element, the properties required are evaluated at the inlet temperatures (which are known) for the first iteration, and the iterations are continued until the outlet temperature converges. The examples considered earlier for the CHTC case are solved again with the VHTC case. The performance parameters are evaluated from equations (11)–(14). They are compared with results of CHTC in Table 3. The pressure drop in every element is calculated using equation (15), with the properties of the fluids evaluated at the local bulk mean temperatures. The elemental pressure drops are summed to get the overall pressure drops in the exchanger. They are presented in Table 4, for comparison with CHTC cases.

The total quantity of heat transferred in VHTC is higher than in the corresponding CHTC cases. The effect of the increased heat transfer is shown in effectiveness ε in Table 3. However, the *NTU*s are found to be lower. Hence, if the ε–*NTU* curve for a particular R is plotted for the VHTC case, it will be found to lie above the corresponding curve for the CHTC case. The *NTU*s for the VHTC cases differ from CHTC cases by a maximum of 6%. Similarly, the εs and θs also differ up to 4 and 10%, respectively. The value of R remains constant. The variation in μ and k reduces the overall heat transfer coefficient, thus reducing *NTU*. But the true mean temperature increases, thus enhancing the heat transferred in the VHTC case.

It can be observed from Table 4 that the pressure drop is higher if the absolute inlet pressures are low. From the results presented in Table 4, it can be observed that the pressure drops in VHTC cases are higher than the pressure drops in CHTC cases by up to 14% for the cold fluid, and lower for the VHTC case by about 3% for the hot fluid.

In most cases, even a large variation in some physical quantities will be reflected only marginally in the performance parameters. Hence, to give a physical feeling of how the temperature distribution is affected by the VHTC assumption, the outlet temperatures obtained from both VHTC and CHTC cases are presented in Table 5.

Stability of the method

To ensure the stability of the method when the number of divisions is small, a constant heat transfer coefficient problem (case 1 in Table 3) is run with a different number of elements. The number of divisions along the hot and cold fluid flow lengths were varied; the performance parameters obtained are presented in Table 6. It is observed that if the number of divisions along the hot and cold fluid lengths are above

Table 5. Outlet temperature comparison

No.	Fluid 1 (hot) Inlet	Fluid 1 (hot) Outlet Constant	Fluid 1 (hot) Outlet Variable	Fluid 2 (cold) Inlet	Fluid 2 (cold) Outlet Constant	Fluid 2 (cold) Outlet Variable
1	355.5	164.4	173.8	56.1	261.6	256.6
2	648.8	250.5	272.2	65.5	480.5	472.7
3	648.8	331.1	352.2	65.5	562.2	546.6
4	648.8	315.5	337.2	65.5	530.0	516.1
5	648.8	390.5	409.4	65.5	605.0	585.0
6	648.8	382.7	398.8	65.5	619.4	603.3
7	648.8	291.1	313.3	65.5	560.5	545.5
8	648.8	230.5	252.2	65.5	499.4	491.1
9	648.8	283.3	304.4	65.5	540.5	528.3

Table 6. The results of experiments with a different number of elements for case 1 of Table 2

No.	Number of longitudinal divisions		Performance parameters		
	Hot fluid	Cold fluid	*NTU*	ε	θ [6]
1	1	1	2.801	0.752	0.268
2	2	1	2.570	0.722	0.281
3	4	2	2.287	0.697	0.305
4	8	4	2.125	0.688	0.324
5	8	5	2.101	0.687	0.327
6	16	12	2.018	0.685	0.339

8 and 5, respectively, the predicted values approach the exact values.

CONCLUSIONS

(1) The finite element model introduced in the present report for the simple crossflow type of compact heat exchangers for the CHTC cases predicts performance parameters which are in good agreement with analytical results.

(2) The model can also be used to predict the performance parameters in VHTC cases.

(3) The model automatically predicts the metal wall temperature distribution along with the temperature distribution of the fluid.

(4) The model can be effectively used for computer-aided rating or sizing of such exchangers.

(5) The present method can be extended to analyse a network of heat exchangers having different heat transfer coefficients and effectivenesses.

(6) There is an effect on the performance of an exchanger when the variation of the heat transfer coefficient throughout the exchanger is taken into account.

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EVALUATION DES PERFORMANCES DES ECHANGEURS DE CHALEUR COMPACTS A ECOULEMENT CROISE AVEC DES ELEMENTS FINIS

Résumé—On analyse l'échangeur de chaleur compact à écoulement croisé en utilisant un modèle à éléments finis. Les prédictions sont en bon accord avec des solutions analytiques disponibles pour les cas de coefficients de transfert thermique constants. L'analyse est étendue au cas du coefficient de transfert variable et les résultats sont comparés aux cas précédents. En outre, on calcule les pertes de charge dans tous les cas.

BERECHNUNG DES VERHALTENS VON KREUZSTROM-KOMPAKT-WÄRMETAUSCHERN MIT HILFE FINITER ELEMENTE

Zusammenfassung—Unter Verwendung finiter Elemente wird ein Kreuzstrom-Kompakt-Wärmetauscher berechnet. Die Ergebnisse stimmen gut mit analytischen Lösungen für Fälle mit konstantem Wärmedurchgangskoeffizienten überein. Die Berechnung wird auf variable Wärmedurchgangskoeffizienten ausgedehnt, die Ergebnisse werden mit Fällen bei konstantem Wärmedurchgangskoeffizienten verglichen. Außer dem thermischen Verhalten wird stets auch der Druckabfall berechnet.

ОЦЕНКА РАБОЧИХ ХАРАКТЕРИСТИК КОМПАКТНЫХ ТЕПЛООБМЕННИКОВ С ПЕРЕКРЕСТНЫМ ПОТОКОМ МЕТОДОМ КОНЕЧНЫХ ЭЛЕМЕНТОВ

Аннотация—С использованием метода конечных элементов анализируются характеристики компактного теплообменника с перектрестным потоком. Расчеты хорошо согласуются с имеющимися аналитическими решениями для случаев постоянного коэффициента теплопереноса. Анализируются случаи переменного коэффициента теплопереноса; результаты сравниваются со случаями постоянного коэффициента. Рассчитываются также перепады давления во всех случаях.